

Replication in Consistent Binomial Models

Christoph Wöster

Bielefeld University

November 2005

BIELEFELD UNIVERSITY

Department of Business

Administration and Economics

P.O. Box 10 01 31

D-33501 Bielefeld

Germany

Discussion Paper No. 545

Replication in Consistent Binomial Models

Christoph Wöster

Department of Business
Administration and Economics
Bielefeld University
P.O. Box 10 01 31
33 501 Bielefeld
Germany
cwoester@uni-bielefeld.de

November 2005

Abstract

The binomial model has been used to price a wide variety of equity and interest rate options for more than two decades. Originally developed by Cox, Ross, and Rubinstein to clarify the basic pricing principle of its continuous-time counterpart with reduced mathematical requirements, the approach became a numerical scheme to evaluate all kinds of contingent claims. Some of the algorithms have dissociated more and more from the basic principles. In this paper we turn to the foundations of the binomial model and elaborate the relation between real world processes, replicating strategies and martingales in a strict way.

Keywords: binomial model, martingale method, option pricing, trading strategy

JEL Classification: G13, C60

1 Introduction

Two perspectives on the nature of the binomial model are presented in this paper. First, the binomial model can be understood as a stand-alone pricing model, whose paths imitate the price behavior of a traded asset in a simplified way. If we start modelling the real world behavior and interpret the paths as the result of a particular state of the world, then we can determine a unique trading strategy that replicates an arbitrary contingent claim. If the law of one price holds, the price of this contingent claim must be equal to the market price of the replicating portfolio. According to the methodology introduced by HARRISON AND KREPS [3], it is possible to formulate the stock process in units of a numeraire (a money market fund) and to create a probability measure under which the process is a martingale. The price of any contingent claim — formulated in units of a numeraire — can be expressed as an expected value of its normed future payoffs under the same measure.

The other interpretation connects the binomial model with a related model, which is assumed to reflect the true processes for theoretical reasons or which simply behaves more like true price paths according to experience. In this context, the binomial model is applied as a numerical scheme to approximate the more adequate model. Since it is only an approximation, some important properties get lost. On the other hand, the reduction of complexity is sometimes the only way to obtain solutions. The binomial model is used to approximate Brownian motions or transformations of the process justified by the central limit theorem. The paths of the binomial model do not play the decisive role any longer; it is the distributional behavior and the convergence property that is of major interest.

In this paper the focus is not on the properties of the binomial model as an algorithmic tool for pricing derivatives; neither the valuation of a new contract nor the analysis of convergence are presented. The question we try to answer is what we can say about the relation between real world processes and martingales if we take the theoretical background and the assumptions of binomial model seriously. Most numerical schemes start modelling the basic processes under the martingale measure and do not bother about real world processes. But it is the real world process that determines the payoff of a derivative and martingales should reflect the replicating strategy.

2 The Valuation Framework

It is assumed that the participants of a financial market have clear and homogenous ideas on the price evolution of some securities (basis securities). In accordance with the modelling of COX, ROSS, AND RUBINSTEIN [2], the future prices are expressed as the outcome of a binomial process. Each path can be associated with an element in the sample space Ω . Together with a σ -Algebra \mathcal{A} and a probability measure P it forms a probability space (Ω, \mathcal{F}, P) . The probability space is equipped with a filtration $\{\mathcal{F}_{t_n}\}_{n=0}^N$ which has the characteristic property

$$\mathcal{F}_{t_0} \subseteq \mathcal{F}_{t_1} \subseteq \dots \subseteq \mathcal{F}_{t_N} = \mathcal{F}$$

representing the evolution of information on the market, where no piece of information gets lost over time.

Trading only takes place at certain equidistant points in time contained in the set

$$\begin{aligned} \mathcal{T} &= \{0 = t_0, t_1, \dots, t_N = T\} \\ &= \{t_0, t_0 + \Delta_t, \dots, t_0 + N \cdot \Delta_t\} \end{aligned}$$

with the overall time interval from 0 to T being fixed. Suppose that the price of the money market fund is determined by the non-stochastic one-period interest rate $r \geq 0$, such that its price evolution can be described by

$$B_{t_n} = \begin{cases} B_{t_0}, & \text{if } t_n = t_0; \\ B_{t_{n-1}} \exp(r\Delta_t), & \text{if } t_0 < t_n \leq t_N. \end{cases} \quad (1)$$

The stochastic process that governs the evolution of the stock price is given by

$$S_{t_n} = \begin{cases} S_{t_0}, & \text{if } t_n = t_0; \\ S_{t_{n-1}} \exp(\mu\Delta_t + \sigma \sqrt{\Delta_t} X_{t_n}), & \text{if } t_0 < t_n \leq t_N; \end{cases} \quad (2)$$

where X_{t_n} is a sequence of independently identically distributed (i.i.d.) Bernoulli random variables

$$X_{t_n} : (\Omega_{t_n}, \mathcal{F}_{t_n}) \rightarrow (\mathcal{X}_{t_n}, \mathcal{B}_{t_n})$$

with outcomes in the state space $\mathcal{X}_{t_n} = \{-1, 1\}$. Given the information at t_n , the probability that $X_{t_{n+1}} = 1$ is p , $0 < p < 1$, and that $X_{t_{n+1}} = -1$ is $(1 - p)$. The parameter $\mu \in \mathbb{R}$ is referred to as the drift coefficient and the parameter $\sigma > 0, \sigma \in \mathbb{R}$, as the diffusion coefficient of the process.

The market is complete by construction. Furthermore, it is assumed that the market is arbitrage-free and remains arbitrage-free after introducing new assets or contracts. A necessary condition for the market being arbitrage-free is

$$\mu - \frac{\sigma}{\sqrt{\Delta_t}} < r < \mu + \frac{\sigma}{\sqrt{\Delta_t}}, \quad (3)$$

i.e. no asset price process dominates the other one.

Our aim is to formulate consistent binomial option pricing models. We give a definition how consistency is understood in this context.

Definition 2.1 (consistent binomial model) *Consider an arbitrage-free binomial model where the expected value of the payoff of an arbitrary derivative expressed in units of a numeraire equals the price of a replicating trading strategy based on real world processes. The model is consistent with respect to parameters $(\hat{\mu}, \hat{\sigma})$ if the expected real world logarithmic return over an interval of length T equals $\hat{\mu}T$ and the corresponding variance is $\hat{\sigma}^2 T$.*

The question we try to answer is how to specify the parameters of the processes under Q such that the real world processes have exactly the distributional properties of the desired kind.

3 Process Parameters and Moments of Logarithmic Returns

Under the specified assumptions the local expected value of the logarithmic return equals

$$\mathbb{E}_P \left[\ln \left(\frac{S_{t_n}}{S_{t_{n-1}}} \right) \middle| \mathcal{F}_{t_{n-1}} \right] = \mu \Delta_t - (1 - 2p) \sigma \sqrt{\Delta_t}$$

and the local variance of the logarithmic return equals

$$\text{Var}_P \left[\ln \left(\frac{S_{t_n}}{S_{t_{n-1}}} \right) \middle| \mathcal{F}_{t_{n-1}} \right] = 4p(1 - p) \sigma^2 \Delta_t.$$

Note that the diffusion parameter σ has no influence on the local expected value if and only if p equals $\frac{1}{2}$. In this case the local variance reduces to

$$\text{Var}_P \left[\ln \left(\frac{S_{t_n}}{S_{t_{n-1}}} \right) \middle| \mathcal{F}_{t_{n-1}} \right] = \sigma^2 \Delta_t.$$

Equal probabilities play a prominent role when interpreting μ and σ as distribution coefficients in a binomial model. Therefore, we frequently split a probability p into a reference probability of $\frac{1}{2}$ and a resulting deviation according to

$$p = \frac{1}{2} + \frac{1}{2}\eta_p, \quad \eta_p \in (-1, 1),$$

which leads to

$$\mathbb{E}_P \left[\ln \left(\frac{S_{t_n}}{S_{t_{n-1}}} \right) \middle| \mathcal{F}_{t_{n-1}} \right] = \mu \Delta_t + \eta_p \sigma \sqrt{\Delta_t}$$

and

$$\text{Var}_P \left[\ln \left(\frac{S_{t_n}}{S_{t_{n-1}}} \right) \middle| \mathcal{F}_{t_{n-1}} \right] = (1 - \eta_p^2) \sigma^2 \Delta_t.$$

For the fixed period from 0 to T one obtains an expected return of

$$\mathbb{E}_P \left[\ln \left(\frac{S_T}{S_0} \right) \middle| \mathcal{F}_0 \right] = N \left(\mu \Delta_t + \eta_p \sigma \sqrt{\Delta_t} \right) = \mu T + \eta_p \sigma \sqrt{N \cdot T} \quad (4)$$

and a variance of

$$\text{Var}_P \left[\ln \left(\frac{S_T}{S_0} \right) \middle| \mathcal{F}_0 \right] = N (1 - \eta_p^2) \sigma^2 \Delta_t = (1 - \eta_p^2) \sigma^2 T. \quad (5)$$

The expected value does not depend on the number of trading days if and only if $\eta_p = 0$, which again underlines the importance of this specification. So far η_p has been interpreted as the spread between the probability of an up move and a down move. Note that the square of η_p has a meaning as well. It can be interpreted as the percentage deviation of the variance of the logarithmic returns from $\sigma^2 T$.

Let us assume that the expected value and the variance of the logarithmic return of the true stock process are known and denoted by $\hat{\mu}$ and $\hat{\sigma}^2$. If the binomial model is interpreted as a

stand-alone model in the sense of the introductory section, then first of all the probability space is specified. The first two moments of the stock's logarithmic returns are then determined by all possible paths implied by the probability space. There is no degree of freedom to calibrate the moments. It is, of course, possible to specify the probability space such that the moments are met — this is the procedure used in practice. However, the probability measure is not really taken into account except for identifying the paths that occur with positive probability. This approach deals with paths and not with moments or distributions.

If we look at the second interpretation, then we are just interested in fitting some properties of the underlying distribution. If we concentrate on the first two moments, the parameters μ , σ and p offer one degree of freedom. Equations (4) and (5) show that the parameters μ and σ^2 can only be interpreted as the moments of the logarithmic returns, if $p = \frac{1}{2}$. Then we get

$$\mu T = \hat{\mu} T \text{ and } \sigma^2 T = \hat{\sigma}^2 T.$$

We can, of course, choose an arbitrary transition probability $p = \frac{1}{2} + \frac{1}{2}\eta_p$ and set

$$\mu = \hat{\mu} - \eta_p \frac{\sigma}{\sqrt{\Delta_t}} \quad (6)$$

and

$$\begin{aligned} \sigma &= \frac{1}{\sqrt{1 - \eta_p^2}} \hat{\sigma} \\ &= \sqrt{\hat{\sigma}^2 + (\hat{\mu} - \mu)^2 \Delta_t} \end{aligned} \quad (7)$$

to obtain the given moments. Putting (7) into (6) yields

$$\mu = \hat{\mu} - \frac{\eta_p}{\sqrt{1 - \eta_p^2}} \frac{\hat{\sigma}}{\sqrt{\Delta_t}}. \quad (8)$$

Solving for η_p^2 one obtains

$$\eta_p^2 = \frac{(\hat{\mu} - \mu)^2}{(\hat{\mu} - \mu)^2 + \frac{\hat{\sigma}^2}{\Delta_t}}. \quad (9)$$

This expression approves to be extraordinarily useful for determining consistent models. It

allows calculating the transition probabilities for a given drift coefficient, such that the (real) distributional parameters are met.

Using (7) and (8) the real world stock price process can be specified as

$$S_{t_n} = \begin{cases} S_{t_0}, & \text{if } t_n = t_0; \\ S_{t_{n-1}} \exp \left(\left(\hat{\mu} - \frac{\eta_p}{\sqrt{1-\eta_p^2}} \hat{\sigma} \right) \Delta_t + \frac{1}{\sqrt{1-\eta_p^2}} \hat{\sigma} \sqrt{\Delta_t} X_{t_n} \right), & \text{if } t_0 < t_n \leq t_N; \end{cases}$$

or equivalently as

$$S_{t_n} = \begin{cases} S_{t_0}, & \text{if } t_n = t_0; \\ S_{t_{n-1}} \exp \left(\hat{\mu} \Delta_t - \hat{\sigma} \sqrt{\Delta_t} X_{t_n}^{\eta_p} \right), & \text{if } t_0 < t_n \leq t_N; \end{cases}$$

with

$$X_{t_n}^{\eta_p} := \frac{X_{t_n} - \eta_p}{\sqrt{1 - \eta_p^2}},$$

having the property that for any arbitrary transition probability $p = \frac{1}{2} + \frac{1}{2}\eta_p$, $0 < p < 1$, its logarithmic return over the time interval $[0, T]$ has mean $\hat{\mu}T$ and variance $\hat{\sigma}^2 T$.

4 Replicating Trading Strategies

A contingent claim is a contract whose payoff structure depends on the state dependent prices of one or more other assets. It is attainable if its future payoff can be generated by a dynamic portfolio strategy. A dynamic portfolio strategy is a previsible process $\phi_{t_n}, n = 0, \dots, N$, representing the number of basis securities that are held in each of the periods $[t_{n-1}, t_n)$. Since the market is complete, the payoff structure of any security contract can be generated by a dynamic portfolio strategy.

In the binomial model the dynamic portfolio strategy is determined by

$$\begin{pmatrix} S_{t_n}(+1) & B_{t_n} \\ S_{t_n}(-1) & B_{t_n} \end{pmatrix} \begin{pmatrix} \phi_{t_n}^S \\ \phi_{t_n}^B \end{pmatrix} = \begin{pmatrix} \Pi_{t_n}(+1) \\ \Pi_{t_n}(-1) \end{pmatrix}, \quad (10)$$

where Π_{t_n} is the price of a portfolio that replicates the (dynamic) payoff structure of the contract

to be valued. The price of this replicating portfolio in t_{n-1} is then given by

$$\Pi_{t_{n-1}} = \phi_{t_n}^S S_{t_{n-1}} + \phi_{t_n}^B B_{t_{n-1}}.$$

We follow the martingale method by HARRISON AND KREPS [3] and express the prices in units of a numeraire by premultiplying both sides of equation (10) by the matrix

$$\mathbf{D}_{t_n} = \begin{pmatrix} B_{t_n}^{-1} & 0 \\ 0 & B_{t_n}^{-1} \end{pmatrix}.$$

Let \hat{X}_{t_n} denote the price of an arbitrary asset or contract X in units of the numeraire, i.e.

$$\hat{X}_{t_n} = B_{t_n}^{-1} X_{t_n}.$$

The number of assets is given by

$$\begin{pmatrix} \phi_{t_n}^S \\ \phi_{t_n}^B \end{pmatrix} = \begin{pmatrix} \frac{\hat{\Pi}_{t_n}(+1) - \hat{\Pi}_{t_n}(-1)}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)} \\ \frac{\hat{S}_{t_n}(+1)\hat{\Pi}_{t_n}(-1) - \hat{S}_{t_n}(-1)\hat{\Pi}_{t_n}(+1)}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)} \end{pmatrix}$$

which results in a portfolio price in t_{n-1} of

$$\begin{aligned} \hat{\Pi}_{t_{n-1}} &= \frac{\hat{\Pi}_{t_n}(+1) - \hat{\Pi}_{t_n}(-1)}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)} \hat{S}_{t_{n-1}} + \frac{\hat{S}_{t_n}(+1)\hat{\Pi}_{t_n}(-1) - \hat{S}_{t_n}(-1)\hat{\Pi}_{t_n}(+1)}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)} \\ &= \frac{\hat{S}_{t_n}(+1) - \hat{S}_{t_{n-1}}}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)} \hat{\Pi}_{t_n}(-1) + \frac{\hat{S}_{t_{n-1}} - \hat{S}_{t_n}(-1)}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)} \hat{\Pi}_{t_n}(+1). \end{aligned}$$

It is well known that the normed price of the replicating portfolio in t_{n-1} can be formulated as the weighted normed prices of the replicating portfolio in t_n and that these weights have the properties of probabilities. Moreover, there is a one-to-one relation between each of the portfolio weights and these pseudo-probabilities, since

$$\phi_{t_n}^S = q \frac{\hat{\Pi}_{t_n}(+1) - \hat{\Pi}_{t_n}(-1)}{\hat{S}_{t_{n-1}} - \hat{S}_{t_n}(-1)}$$

and

$$\phi_{t_n}^B = q \frac{\hat{S}_{t_n}(+1)\hat{\Pi}_{t_n}(-1) - \hat{S}_{t_n}(-1)\hat{\Pi}_{t_n}(+1)}{\hat{S}_{t_{n-1}} - \hat{S}_{t_n}(-1)}.$$

The structure of

$$q := \frac{\hat{S}_{t_{n-1}} - \hat{S}_{t_n}(-1)}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)}$$

if asset price processes are modelled according to (1) and (2) is another point we want to record here. For the specified processes the weight q must be

$$q = \frac{\exp(-(\mu - r)\Delta_t) - \exp(-\sigma \sqrt{\Delta_t})}{\exp(\sigma \sqrt{\Delta_t}) - \exp(-\sigma \sqrt{\Delta_t})} \quad (11)$$

$$= \frac{1}{2} + \frac{\exp(-(\mu - r)\Delta_t) - \cosh(\sigma \sqrt{\Delta_t})}{2 \sinh(\sigma \sqrt{\Delta_t})} \quad (12)$$

to reflect a replicating trading strategy.

5 Martingale processes

5.1 Parameter restrictions

The aim is to find a martingale measure under which the normed prices of all traded assets are martingales. We follow the standard procedure and formulate the prices of the stock in units of the money market fund. We obtain

$$\begin{aligned} \hat{S}_{t_n} &:= B_{t_n}^{-1} S_{t_n} \\ &= \hat{S}_{t_0} \exp \left((\alpha - r)(t_n - t_0) + \beta \sqrt{\Delta_t} \sum_{i=1}^n X_{t_i} \right). \end{aligned}$$

Obviously, the expected value exists and hence, the process is a martingale if the following condition is satisfied:

$$\begin{aligned} \mathbb{E}_Q [\hat{S}_{t_{n+1}} | \mathcal{F}_{t_n}] &= B_{t_{n+1}}^{-1} \left(q S_{t_n} \exp(\alpha \Delta_t + \beta \sqrt{\Delta_t}) + (1 - q) S_{t_n} \exp(\alpha \Delta_t - \beta \sqrt{\Delta_t}) \right) \\ &\stackrel{!}{=} B_{t_n}^{-1} S_{t_n} = \hat{S}_{t_n}. \end{aligned} \quad (13)$$

This is equivalent to

$$\exp(-(\alpha - r)\Delta_t) = \left[q \exp(\beta \sqrt{\Delta_t}) + (1 - q) \exp(-\beta \sqrt{\Delta_t}) \right]$$

which can be analyzed more conveniently if we split up the transition probability q induced by the martingale measure according to

$$q = \frac{1}{2} + \frac{1}{2}\eta_q.$$

The expression simplifies to

$$\exp(-(\alpha - r)\Delta_t) = \left(\cosh(\beta \sqrt{\Delta_t}) + \eta_q \sinh(\beta \sqrt{\Delta_t}) \right)$$

and results in a condition for

$$\eta_q = \frac{\exp(-(\alpha - r)\Delta_t) - \cosh(\beta \sqrt{\Delta_t})}{\sinh(\beta \sqrt{\Delta_t})}.$$

The quantity q is a probability if

$$\alpha - \frac{\beta}{\sqrt{\Delta_t}} < r < \alpha + \frac{\beta}{\sqrt{\Delta_t}}$$

is satisfied. A detailed discussion of the relationship of drift parameters and transition probabilities in arbitrage-free binomial models can be found in WÖSTER [7].

5.2 Replicating a given real world process

This interpretation starts with a given probability space (Ω, \mathcal{F}, P) describing the states of the real world. Since the probability measure P is given, the transition probabilities and henceforth the probability spread η_p is determined. If the stock price evolves according to (2), then for a given mean $\hat{\mu}$ and a given variance $\hat{\sigma}^2$ the drift coefficient μ and the diffusion coefficient σ are determined by (6) and (7).

Given the real world probability space and the given structure of the basic asset price processes, there is a unique trading strategy replicating the payoff of a specified contract. Since the payoff of a derivative is determined by the real world stock process, we have to choose

$$\beta = \sigma = \frac{1}{\sqrt{1 - \eta_p^2}} \hat{\sigma}$$

and

$$\alpha = \mu = \hat{\mu} - \frac{\eta_p}{\sqrt{(1 - \eta_p^2)}} \frac{\hat{\sigma}}{\sqrt{\Delta_t}}.$$

The martingale measure is then given by

$$q = \frac{1}{2} + \frac{\exp(-(\mu - r)\Delta_t) - \cosh(\sigma \sqrt{\Delta_t})}{2 \sinh(\sigma \sqrt{\Delta_t})} \quad (14)$$

provided that condition (3) holds. A comparison with (11) shows that the transition probability q given by (14) reflects a replicating trading strategy.

5.3 The real world background process of a given martingale process

The standard procedure is to model martingales without thinking about the real world processes that back them, i.e. those real world processes on which replicating strategies are based. In this section we analyze approaches starting with martingales.

There are two main procedures to obtain the martingale property of a binomial process. The first one is to fix a transition probability q and the diffusion parameter β under the martingale measure. The drift parameter is then uniquely determined by

$$\alpha(q, \beta) = r - \frac{1}{\Delta_t} \ln \left(\cosh \left(\beta \sqrt{\Delta_t} + \eta_q \sinh \left(\beta \sqrt{\Delta_t} \right) \right) \right).$$

A very convenient choice is $q = \frac{1}{2}$, which results in

$$\alpha\left(\frac{1}{2}, \beta\right) = r - \frac{1}{\Delta_t} \left(\cosh \left(\beta \sqrt{\Delta_t} \right) \right).$$

Equal transition probabilities have been applied to stock processes in option pricing models by AMIN [1] and to interest rate processes in term structure by JARROW [6] and HEATH, JARROW, AND MORTON [4, 5].

The second approach fixes the drift parameter α and the diffusion parameter β under the martingale measure. The corresponding transition probability must be

$$q(\alpha, \beta) = \frac{1}{2} + \frac{\exp(-(\alpha - r)\Delta_t) - \cosh(\beta \sqrt{\Delta_t})}{2 \sinh(\beta \sqrt{\Delta_t})}$$

under the given parameter constellation. This method was proposed by COX, ROSS, AND RUBINSTEIN [2] setting $\alpha = 0$.

The question arises how to determine parameter β . It is often argued that β should correspond to the standard deviations of real world logarithmic returns. However, $\beta = \hat{\sigma}$ implies

$$\sigma = \frac{\beta}{\sqrt{1 - \eta_p^2}}.$$

This in turn means the process under the risk neutral measure does not reflect trading strategies under the real world measure unless the transition probability equals $\frac{1}{2}$. The problem is that the real world distributional behavior (and hence the standard deviation $\hat{\sigma}$) depends on the real world measure which is typically never specified in these binomial model implementations.

There is a consistent way to specify the parameters of a martingale process for fixed transition probabilities that retains the fundamental principle of replicating payoffs in the real world. The solution is to set $\alpha = \mu$ and $\beta = \sigma$. Under these specifications it is consistent to calculate the derivative on the dynamics of the martingale asset price. Given a drift coefficient α , the diffusion coefficient is determined by (8) to be compatible with predefined distribution parameters. Thus, the coefficient reads

$$\beta = \sqrt{(\hat{\mu} - \alpha)^2 \Delta + \hat{\sigma}^2}.$$

On the one hand the difference between β and $\hat{\sigma}$ is very small in practice and there seems to be no need for an adjustment, on the other hand this correction is easily done.

Of course, the measure of the real world probabilities have to be adjusted. The probability spread can be computed according to (9) and results in

$$\eta_p = \frac{\hat{\mu} - \alpha}{\sqrt{(\hat{\mu} - \alpha)^2 + \frac{\hat{\sigma}^2}{\Delta_t}}}. \quad (15)$$

6 An Example: An Up-and-Out Binary Option

It is assumed that a money market fund can be purchased promising a fixed interest rate quoted as an instantaneously compounded rate

$$r = 0.06.$$

Moreover, the expected logarithmic returns of the stock and their corresponding variances are independent of time and known with certainty. Suppose that the values are given by

$$\hat{\mu} = 0.12$$

and

$$\hat{\sigma}^2 = 0.09.$$

As we know from section 3, the stock price process must be of the structure

$$S_{t_n} = \begin{cases} S_{t_0}, & \text{if } t_n = t_0; \\ S_{t_{n-1}} \exp \left(\left(0.12 - \frac{\eta_p}{\sqrt{1-\eta_p^2}} 0.3 \right) \Delta_t + \frac{1}{\sqrt{1-\eta_p^2}} 0.3 \sqrt{\Delta_t} X_{t_n} \right), & \text{if } t_0 < t_n \leq t_N. \end{cases}$$

In t_0 the price of one share of the money market fund is normalized to a price of 1, the stock quotes at a price of 20.

Suppose the arbitrage-free price of an up-and-out stock-or-nothing binary option with a hurdle $H = 24$ is to be determined. The contract matures in 0.25 time units. There are three equidistant trading days (excluding the current day, including the maturity day), so $\Delta_t = \frac{1}{12}$. The option's payoff at maturity $C_{t_N}(H)$ is given by

$$C_{t_3}(24) = \begin{cases} S_{t_3}, & S_{t_n} < 24 \forall t_n \in \mathcal{T}; \\ 0, & \text{otherwise.} \end{cases}$$

in this example. The contract does not pay any money before maturity.

6.1 Security Processes Under the Real World Measure \mathbf{P}

Let us start with the processes representing the security price behavior in the real world. Suppose that the probability for an up move equals the probability for a down move. These transition probabilities have been assigned to the edges of the tree in figure 1. We have to stress that they are neither necessary to build a replicating trading strategy nor to determine the value of an option using the martingale method.

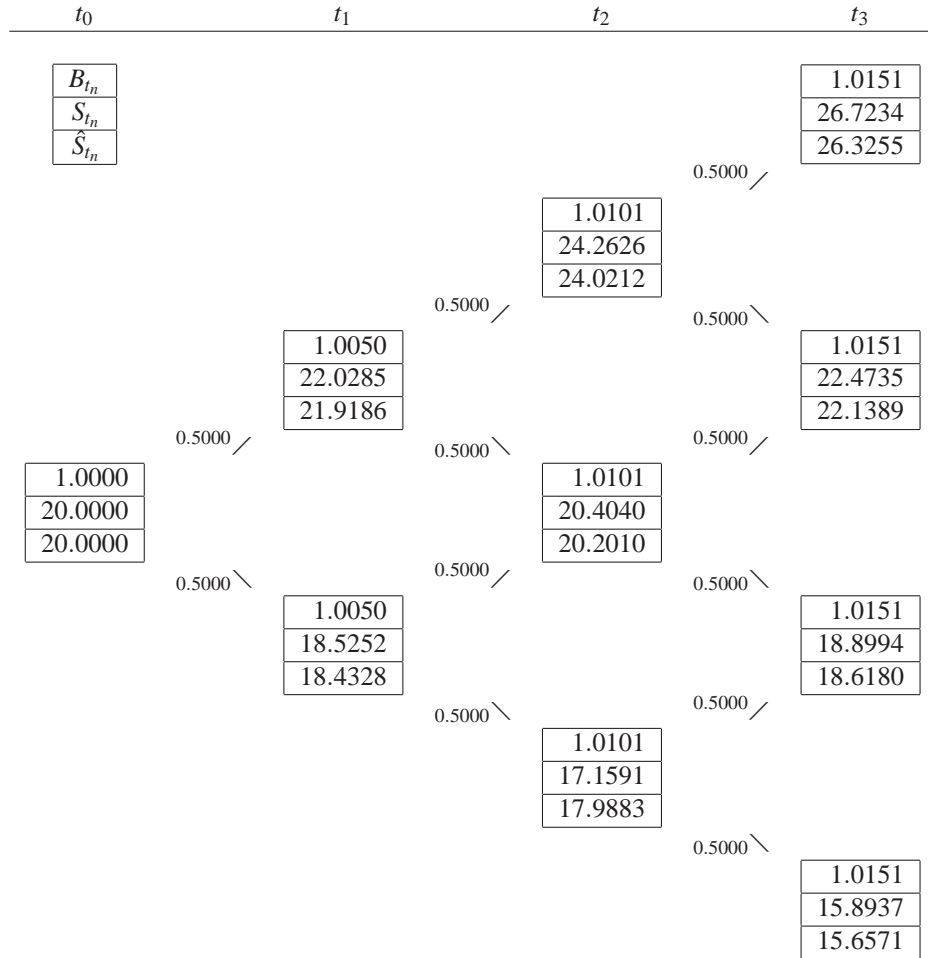


Figure 1: Binomial tree under the real world probability measure

However, the transition probabilities are implicitly determined if the distributional parameters $(\hat{\mu}, \hat{\sigma})$ and the process parameters (μ, σ) are given. Since we wish to have the equality of $\hat{\mu}$ and μ on the one hand and $\hat{\sigma}$ and σ on the other hand we have to choose a transition probability $p = \frac{1}{2}$.

The last entries refer to the stock prices in units of the numeraire. In this case the money market fund has been used as the reference quantity.

6.2 The Replicating Portfolio Strategy

If we assume that the processes formed in the previous subsections reflect the possible outcome of the processes given the real world probability space, then the trading strategy formulated in figure 2 is the unique portfolio process generating the payoff of the binary option.

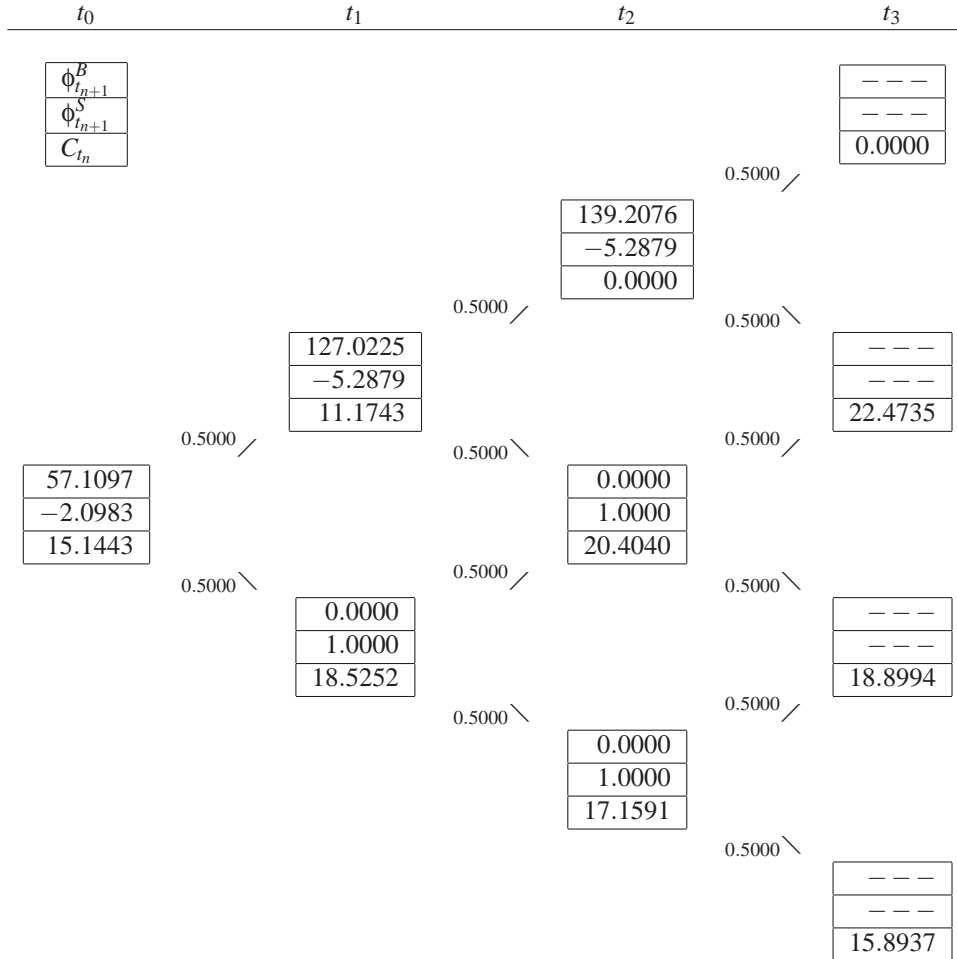


Figure 2: Replicating strategy for the binary option

The trading strategy is determined recursively by the formulae introduced in section 4. If the payoff at maturity is not equal to 0, then it must be equal to the corresponding stock price. The standard procedure is to calculate the value of the option at a certain time independently of the stock price up to this time. The continuation value is replaced by 0 if the stock price at the corresponding node hits or exceeds the hurdle H . This is the case at time t_2 in the uppermost node.

6.3 Security Processes Under the Martingale Measure Q

Now our view will change to the so-called risk neutral world where we consider martingale processes rather than real world processes. The first example keeps the real world processes in mind and models martingales that are consistent with the real world described above. The second example is an application of the CRR model specification.

6.3.1 A Model Based on Real World Processes

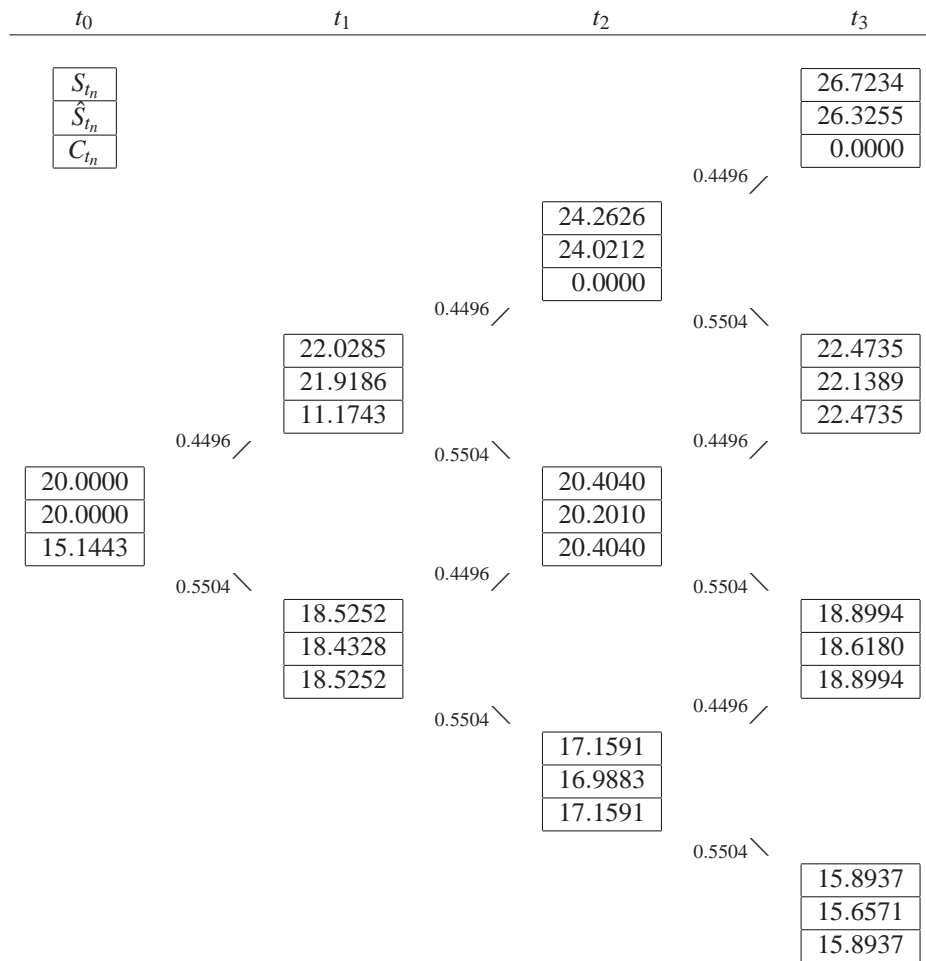


Figure 3: Binomial tree with a drift $\alpha = \mu$

Since the payoff of the derivative for which the price is to be determined depends on the stock price in the real world and not in the risk neutral world, it seems to be a natural approach to retain the dynamics of the real world process, to calculate the derivative payoffs and to change the measure such that the prices in the numeraire are martingales.

Figure 3 shows that the arbitrage-free price process of the derivative is compatible with the price process of the replicating trading strategy in the previous section, especially the prices in t_0 coincide.

6.3.2 The Cox-Ross-Rubinstein Model

Several proposals have been made in the financial literature how to implement binomial option pricing models. The most famous model has been developed by COX, ROSS, AND RUBINSTEIN [2]. The standard way to calibrate this model to the real world data of our example is to set $\alpha = 0$ and

$$\beta = \sigma = 0.3.$$

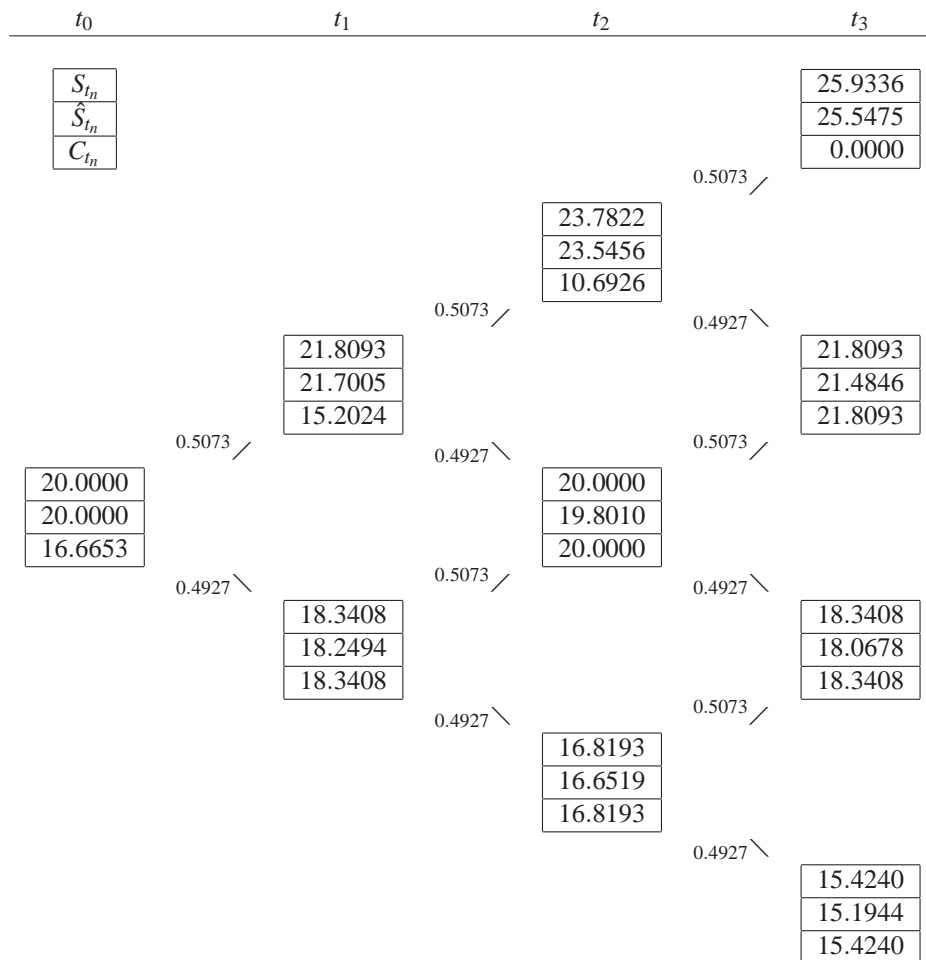


Figure 4: Binomial tree with a drift $\alpha = 0$

This yields a probability q under the martingale measure of

$$q(0, 0.3) = \frac{1}{2} + \frac{\exp\left(\frac{0.06}{12}\right) - \cosh\left(0.3 \sqrt{\frac{1}{12}}\right)}{2 \sinh\left(0.3 \sqrt{\frac{1}{12}}\right)} \approx 0.5073.$$

Note that this process does not reflect the trading strategy of section 4. The arbitrage-free price of the barrier option differs significantly from the price of the replicating strategy. There is a remarkable difference at the uppermost node at time t_2 . Whereas the option value is strictly positive in the CRR model, it is 0 in this state at the same time in the preceding model. Given our real world process the option value must not have a positive value since the price of the underlying is 24.2626 and hence quoting above the hurdle which makes the option worthless. The CRR model has taken the martingale process multiplied by B_{t_2} to check if the stock price has hit or exceeded the hurdle. But this process has nothing in common with our real world processes which are responsible for the derivative payoff. Thus, if the CRR model does not represent our real world process, then what does it represent?

7 Interpreting the Approaches

We have to be careful when interpreting the results of the last sections. At a first glance the model seems to be inadequate to reflect the trading strategies based on real world processes. However, such a conclusion would be overhasty. The right formulation is that there exists a real world process associated with a real world probability measure such that trading strategies based on these processes are reflected by the CRR model.

If we use the stock process having a drift parameter α and a diffusion parameter β to determine the payoff of a derivative contract, then these parameters are not only the coefficients under the martingale measure Q but also under the real world measure P . Now, we can determine the undistorted diffusion coefficient β , which is given by

$$\begin{aligned} \beta &= \sqrt{(\hat{\mu} - \alpha) \Delta_t + \hat{\sigma}^2} \\ &= \sqrt{(0.12 - 0) \cdot \frac{1}{12} + 0.09} = 0.3020. \end{aligned}$$

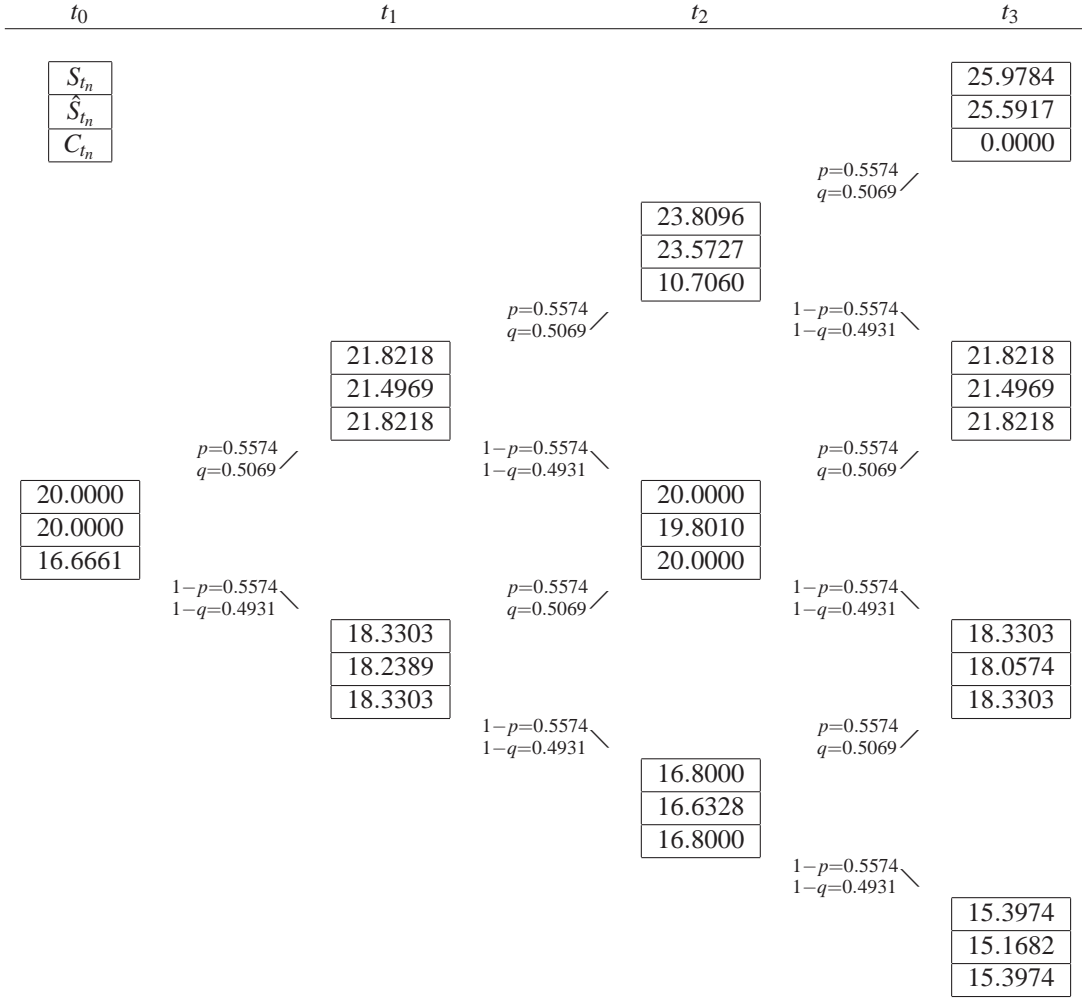


Figure 5: A consistent CRR binomial tree

The deviation from the real world diffusion parameter $\hat{\sigma}$ is very small, even though the time discretization is rather coarse. The more interesting result is that we can identify in straight-forward way the real world process for which the replicating trading strategy is reflected by martingales in the CRR model. The probability spread is given by

$$\begin{aligned} \eta_p &= \frac{\hat{\mu} - \alpha}{\sqrt{(\hat{\mu} - \alpha)^2 + \frac{\hat{\sigma}^2}{\Delta t}}} \\ &= \frac{0.12}{\sqrt{0.12 + 0.09 \cdot 12}} = 0.1147, \end{aligned}$$

and henceforth the transition probability by

$$p = 0.5 + 0.5 \cdot \eta_p = 0.5574.$$

To summarize, the CRR binomial process in figure 5 satisfies three properties: First, all prices in units of the numeraire are martingales. Secondly, the logarithmic returns of the price processes have the predetermined distributional parameters. Thirdly, the processes under the martingale measure reflect replicating trading strategies since the process parameters in both worlds coincide. Hence, the model is consistent in the above mentioned sense.

8 Concluding Remarks

This paper tries to clarify the relation between distributional parameters in the real world, namely the expected value and variance of logarithmic returns, the process parameters in the real world and the process parameters in the risk neutral world.

There is a continuum of process parameters (μ, σ) and transition probabilities p that result in a process with given distributional parameters $(\hat{\mu}, \hat{\sigma})$. The exact relations have been shown in section 3. If we fix the process parameters and the transition probabilities (probably with certain distributional parameters in mind), then the trading strategy of a certain derivative payoff structure is uniquely determined. Furthermore, there is a one-to-one relation between the number of assets held in this strategy and transition probabilities under the martingale measure. Thus, the risk-neutral world is determined as well.

The usual procedure is to model the other way round. The starting point is the stock process in units of a numeraire which is required to be a martingale. However, if the calculation of the derivative payoff is based on this process, then the real world process must have the same structure. In other words, the process parameters of the real world and the risk neutral world have been determined simultaneously, the real world model can then be completed by identifying the probability measure that yields the actual distributional parameters. In general, this means that the martingale diffusion parameter can only be determined after specifying the consistent real world model. However, the distortion that arises if we set the risk neutral diffusion parameter to the standard deviation of the logarithmic returns will be negligibly small in nearly all applications.

References

- [1] Amin, Kaushik I., 1991, On the computation of continuous time option prices using discrete approximations, *Journal of Financial and Quantitative Analysis* 26, 477–495.
- [2] Cox, John C., Stephen A. Ross, and Mark Rubinstein, 1979, Option pricing: A simplified approach, *Journal of Financial Economics* 7, 229–263.
- [3] Harrison, J. Michael, and David M. Kreps, 1979, Martingales and arbitrage in multiperiod securities markets, *Journal of Economic Theory* 20, 381–408.
- [4] Heath, David, Robert A. Jarrow, and Andrew Morton, 1990a, Bond pricing and the term structure of interest rates: A discrete time approximation, *Journal of Financial and Quantitative Analysis* 25, 419–410.
- [5] ———, 1990b, Contingent claim valuation with a random evolution of interest rates, *Review of Futures Markets* 9, 54–76.
- [6] Jarrow, Robert A., 1996, *Modelling Fixed Income Securities and Interest Rate Options* (McGraw-Hill Companies: New York, St. Louis, San Francisco et al.).
- [7] Wöster, Christoph, 2004, Constructing arbitrage-free binomial models, Discussion Paper No. 530, Bielefeld University.

Diskussionspapiere der Fakultät für Wirtschaftswissenschaften

- | | | | | | |
|------|---|---|------|--|--|
| 332. | Ralf-M. Marquardt | Die EU auf dem Weg zur EWWU: Ein wichtiger Fortschritt für den Außenhandel?
Januar 1997 | 346. | Thorsten Hens/
Andreas Löffler | Existence and Uniqueness of Equilibria in the CAPM with a Riskless Asset
Dezember 1995
(Universität Bielefeld, Mai 1997) |
| 333. | Frohn, Chen, Franke, Gottschalk, Jacobebbinghaus, Kräussl, Leuchtmann, Ludden, Oelker, Vollmann | Drei Simulationsexperimente mit dem Bielefelder Modell zur Erfassung der ökonomischen Wirkungen umweltpolitischer Maßnahmen (Einführung des Dualen Systems, Erhöhung des Benzinpreises, CO_2 -Reduktion)
Januar 1997 | 347. | Jean-Marc Bottazzi/
Thorsten Hens/
Andreas Löffler | Market Demand Functions in the CAPM
Dezember 1996
(Universität Bielefeld, Mai 1997) |
| 334. | Ralf-M. Marquardt | Gefährden die Arbeitskosten den Investitionsstandort Deutschland?
März 1997 | 348. | Piero Gottardi/
Thorsten Hens | Disaggregation of Excess Demand and Comparative Statics with Incomplete Markets and Nominal Assets
Mai 1997 |
| 335. | Carl Chiarella
Peter Flaschel | Keynesian monetary growth dynamics in open economies
March 1997 | 349. | Gang Gong | The Multiplier Process in a Temporary General Equilibrium Model
Juni 1997 |
| 336. | Carl Chiarella
Peter Flaschel | The dynamics of 'natural' rates of growth and employment
March 1997 | 350. | M.O. Bettzüge/
Thorsten Hens | An Evolutionary Approach to Financial Innovation
Juli 1997 |
| 337. | Hermann Jahnke/
Anne Chwolka | Strategische Kostenrechnung: Eine spieltheoretische Begriffsbildung
Mai 1997 | 351. | Reinhard John/
Matthias G. Raith | Optimizing Multi-Stage Negotiations
August 1997 |
| 338. | Willi Semmler/
Alfred Greiner | An Inquiry into the Sustainability of German Fiscal Policy: Some Simple Tests
Mai 1997 | 352. | Klaus Reiner Schenk-Hoppé | Evolutionary Stability of Walrasian Equilibria
August 1997 |
| 339. | Willi Semmler/
Levent Kockesen | Testing the Financial Accelerator Using Nonlinear Time Series Methods
Mai 1997 | 353. | Klaus Reiner Schenk-Hoppé | Bifurcations of the Randomly Perturbed Logistic Map
August 1997 |
| 340. | Willi Semmler/
Alfred Greiner | Estimating an Endogenous Growth Model with Public Capital and Government Borrowing
Mai 1997 | 354. | Thorsten Spitta | Standardsoftware zur Verwaltung und Führung von Fakultäten
August 1997 |
| 341. | Leo Kaas | Multiplicity of Cournot Equilibria and Involuntary Unemployment
Mai 1997 | 355. | Volker Böhm/
Nicole Köhler/
Jan Wenzelburger | Endogenous Random Asset Prices In Overlapping Generations Economies
September 1997 |
| 342. | Leo Kaas | Imperfectly Competitive Price Setting under Bayesian Learning in a Disequilibrium Model
Mai 1997 | 356. | Volker Böhm/
Jan Wenzelburger | Perfect Predictions in Economic Dynamical Systems with Random Perturbations
September 1997 |
| 343. | Volker Böhm/
Leo Kaas | Differential Savings, Factor Shares, and Endogenous Growth Cycles
Mai 1997 | 357. | Peter Flaschel | Disequilibrium Growth Theory with Insider/Outsider Effects.
August 1997 |
| 344. | K.R. Schenk-Hoppé | The Evolution of Walrasian Behavior in Oligopolies
April 1997 | 358. | Peter Flaschel | On the Dominance of the Keynesian Regime in Disequilibrium Growth Theory.
August 1997 |
| 345. | Jürgen Krüll | UNIX-Accounting als Datenbasis des IV-Controlling - Möglichkeiten und Grenzen -
Mai 1997 | 359. | Peter Flaschel/
Gangolf Groh | Textbook Stagflation Theory: Narrow views and full implications.
September 1997 |
| | | | 360. | Roman Kräussl | Einführung in RATS
September 1997 |
| | | | 361. | Martin Lettau/
Gang Gong/
Willi Semmler | Statistical Estimation and Moment Evaluation of a Stochastic Growth Model with Asset Market
Oktober 1997 |

Diskussionspapiere der Fakultät für Wirtschaftswissenschaften

- | | | | | | |
|------|---|--|------|---|--|
| 362. | Marc Oliver Bettzüge/
Thorsten Hens/
Marta Laitenberger | On Choquet Prices in a GEI-Model with Intermediation Costs
August 1997 | 379. | Willi Semmler/
Malte Sieveking | The Use of Vector Field Analysis for Studying Debt Dynamics
Januar 1998 |
| 363. | Peter Flaschel/
Rajiv Sethi | Stability of Models of Money and Perfect Foresight: Implications of Nonlinearity
September 1997 | 380. | Dirk Biskup/
Dirk Simons | Game Theoretic Approaches to Cost Allocation in the Dynamic Total Tardiness Problem
Januar 1998 |
| 364. | Peter Flaschel | Keynes-Marx and Keynes-Wicksell models of monetary growth: A framework for future analysis
September 1997 | 381. | Eckart Jäger | Exchange Rates and Bertrand-Oligopoly
Januar 1998 |
| 365. | Peter Flaschel | Corridor stability and viability in economic growth
September 1997 | 382. | Michael J. Fallgatter/
Lambert T. Koch | Zur Rezeption des radikalen Konstruktivismus in der betriebswirtschaftlichen Organisationsforschung
Januar 1998 |
| 366. | Gang Gong | Endogenous Technical Change and Irregular Growth Cycles with Excess Capacity
Oktober 1997 | 383. | Reinhold Decker/
Ralf Wagner | Log-lineare Modelle in der Marktforschung
Januar 1998 |
| 367. | Gang Gong | Growth, Interest Rate and Financial Instability
Oktober 1997 | 384. | Willi Semmler/
Malte Sieveking | External Debt Dynamics and Debt Cycles: The Role of the Discount Rate
Februar 1998 |
| 368. | Thorsten Hens/
Eckart Jäger/
Alan Kirman/
Louis Phlips | Exchange Rates and Oligopoly
Oktober 1997 | 385. | Rolf König/
Michael Wosnitza | Zur Problematik der Besteuerung privater Aktienkursgewinne - Eine ökonomische Analyse
März 1998 |
| 369. | Thorsten Hens/
Karl Schmiedders/
Beate Voß | On Multiplicity of Competitive Equilibria when Financial Markets are Incomplete | 386. | Joachim Frohn | Zum Nutzen struktureller makro-ökonomischer Modelle
April 1998 |
| 370. | Ralf Wagner/
Thorsten Temme/
Reinhold Decker | Auftreten von und Möglichkeiten des Umgangs mit fehlenden Werten in der Marktforschung
Oktober 1997 | 387. | K. R. Schenk-Hoppé/
Björn Schmalfuß | Random Fixed Points in a Stochastic Solow Growth Model, April 1998 |
| 371. | Toichiro Asada/
Willi Semmler/
Andreas J. Novak | Endogenous Growth and the Balanced Growth Equilibrium
November 1997 | 388. | Sandra Güth | Evolution of Trading Strategies
April 1998 |
| 372. | Carlos Alos-Ferrer/
Ana B. Ania/
K.R. Schenk-Hoppé | A dynamic evolutionary model of Bertrand oligopoly
November 1997 | 389. | Hermann Jahnke | Losgrößentheorie und betriebliche Produktionsplanung
April 1998 |
| 373. | Volker Böhm/
Jan Wenzelburger | Expectational Leads in Economic Dynamical Systems
Dezember 1997 | 390. | Ralf-M. Marquardt | Geldmengenkonzept für die EZB? - Ein Mythos als Vorbild
April 1998 |
| 374. | A. Sigge/
Th. Spitta | Die Workbench des Systems R/3 als Beispiel einer Software-Entwicklungsumgebung
Dezember 1997 | 391. | Anne Chwolka | Delegation of Planning Activities and the Assignment of Decision Rights
April 1998 |
| 375. | Carl Chiarella/
Peter Flaschel | An integrative approach to disequilibrium growth dynamics in open economies
Dezember 1997 | 392. | Thorsten Spitta | Schnittstellengestaltung in modularen Unternehmen
Mai 1998 |
| 376. | Joachim Frohn | Ein Marktmodell zur Erfassung von Wanderungen
Dezember 1997 | 393. | Notburga Ott | Zur Konzeption eines Familienlastenausgleichs
Mai 1998 |
| 377. | Dirk Biskup | Single-Machine Scheduling with Learning Considerations
Januar 1998 | 394. | Röhle, M./
Wagner, U./
Decker, R. | Zur methodengestützten Validierung stochastischer Kaufverhaltensmodelle
Mai 1998 |
| 378. | Hermann Jahnke | Produktionswirtschaftliche Steuergrößen, Unicherheit und die Folgen | 395. | Frank Laß | Der neue § 50 c Abs. 11 EStG induziert keine Besteuerung privater Ver- |

Diskussionspapiere der Fakultät für Wirtschaftswissenschaften

- | | | | | | |
|------|---|--|------|--|---|
| | | äußerungsgewinne von Anteilen an Kapitalgesellschaften
Mai 1998 | | | |
| 396. | Dirk Simons | Optimale Ausübungszeitpunkte für Optionen aus Aktienoptionsprogrammen unter Einbeziehung steuerlicher Liquiditätswirkungen
August 1998 | 411. | Dirk Simons | Die Koexistenz von Rechnungslegungsnormen betreffend F&E-Projekte innerhalb der EU und ihr Einfluß auf die Investitionstätigkeit von Eigenkapitalgebern
Februar 1999 |
| 397. | Dirk Biskup/
Martin Feldmann | Benchmarks for scheduling on a single-machine against restrictive and unrestrictive common due dates
August 1998 | 412. | Anne Chwolka/
Matthias G. Raith | Group Preference Aggregation with the AHP - Implications for Multiple-issue Agendas
Februar 1999 |
| 398. | Anton Stiefenhofer | Chaos in Cobweb Models Due to Price Uncertainty
September 1998 | 413. | Jürgen Krüll | Literate System-Administration (LiSA) - Konzept und Erprobung dokumentenbasierten System-managements -
März 1999 |
| 399. | Volker Böhm | Macroeconomic Dynamics with Sequential Trading
September 1998 | 414. | Jürgen Krüll/
Ha-Binh Ly | Literate System-Administration (LiSA) - Konzept und Realisierung einer Arbeitsumgebung für den Systemadministrator -
März 1999 |
| 400. | Volker Böhm/
Klaus Reiner Schenk-Hoppé | MACRODYN - A User's Guide
August 1998 | 415. | Hans Peter Wolf | RREVIVE - Funktionen zur Arbeit mit wiederbelebten Papieren unter R |
| 401. | Thorsten Spitta | Data Collection of Development and Maintenance Effort - Data Model and Experiences
August 1998 | 416. | Volker Böhm | Stochastische Wachstumszyklen aus dynamischer Sicht
März 1999 |
| 402. | Hermann Jahnke/
Anne Chwolka | Preis- und Kapazitätsplanung mit Hilfe kostenorientierter Entscheidungsregeln
September 1998 | 417. | Hermann Jahnke/
Dirk Biskup/
Dirk Simons | The Effect of Capital Lockup and Customer Trade Credits on the Optimal Lot Size – A Confirmation of the EOQ
März 1999 |
| 403. | Reinhold Decker/
Thorsten Temme | Einsatzmöglichkeiten der Diskriminanzanalyse in der Marktforschung
September 1998 | 418. | Peter Naeve,
Hans Peter Wolf,
Lars Hartke,
Ulrich Kirchhoff,
Dirk Tigler | Portierung des REVBOOK nach R für die Digitale Bibliothek NRW – ein Projektbericht
April 1999 |
| 404. | Thorsten Spitta | Grundlagen der Betriebsinformatik - Ein Versuch dsziplinübergreifender Lehre –
Oktober 1998 | 419. | Thorsten Temme,
Reinhold Decker | Analyse a priori definierter Gruppen in der angewandten Marktforschung
März 1999 |
| 405. | Michael J. Fallgatter | Leistungsbeurteilungstheorie und -praxis: Zur „Rationalität“ der Ignorierung theoretischer Empfehlungen
Oktober 1998 | 420. | Klaus Reiner Schenk-Hoppé | Is There A Golden Rule For The Stochastic Solow Growth Model?
März 1999 |
| 406. | K.R. Schenk-Hoppé | Bounds on Sample Paths of Stochastic Nonlinear Systems - A Lyapunov Function Approach
Dezember 1998 | 421. | Gang Gong,
Willi Semmler,
Peter Flaschel | A Macroeconometric Study on the Labor Market and Monetary Policy: Germany and the EMU
Januar 1999 |
| 407. | Hans Peter Wolf | Ein wiederbelebbares Buch zur Statistik, Dezember 1998 | 422. | Peter Flaschel,
Gang Gong,
Willi Semmler | A Keynesian Based Econometric Framework for Studying Monetary Policy Rules, März 1998 |
| 408. | Claudia Bornemeyer/
Thorsten Temme/
Reinhold Decker | Erfolgsfaktorenforschung im Stadtmarketing unter besonderer Berücksichtigung multivariater Analysemethoden
Dezember 1998 | 423. | Carl Chiarella,
Willi Semmler,
Stefan Mittnik | Stock Market, Interest Rate and Output: A Model and Estimation for US Time Series Data
Dezember 1998 |
| 409. | Hans Peter Wolf | Datenanalysen mit algorithmischen Erfordernissen exemplarisch demonstriert anhand einer Untersuchung des Leistungsstands von Studierenden
Januar 1999 | 424. | Martin Feldmann | A Development Framework for Nature Analogic Heuristics |
| 410. | Klaus-Peter Kistner | Lot Sizing and Queueing Models | | | |

Diskussionspapiere der Fakultät für Wirtschaftswissenschaften

		Mai 1999			
425.	Dirk Biskup, Martin Feldmann	Single-machine scheduling for minimizing earliness and tardiness penalties by meta-heuristic approaches Juni 1999	441.	Anne Chwolka, Dirk Simons	Impacts of Revenue Sharing, Profit Sharing, and Transfer Pricing on Quality-Improving Investments Januar 2000
426.	Anne Chwolka	Choice of Information Systems for Decision and Control Problems August 1999	442.	Carsten Köper, Peter Flaschel	Real-Financial Interaction: A Keynes-Metzler-Goodwin Portfolio Approach Januar 2000
427.	Joachim Frohn	Macroeconometric Models versus Vectorautoregressive Models August 1999	443.	Th. Spitta, R. Decker, A. Sigge, P. Wolf, V. Tiemann	Erste Bilanz des Kreditpunktesystems der Fakultät für Wirtschaftswissenschaften Januar 2000
428.	Caren Sureth, Rolf König	General investment neutral tax systems and real options März 1999	444.	Imre Dobos	A dynamic theory of production: flow or stock-flow production Functions Februar 2000
429.	Imre Dobos, Klaus-Peter Kistner	Optimal Production-Inventory Strategies for a Reverse Logistics System Oktober 1999	445.	Carl Chiarella, Peter Flaschel	Applying Disequilibrium Growth Theory: I. Investment, Debt and Debt Deflation January 2000
430.	Dirk Biskup, Hermann Jahnke	Common Due Date Assignment for Scheduling on a Single Machine With Jointly Reducible Processing Times Oktober 1999	446.	Imre Dobos	A Dynamic Environmental Theory of Production Maerz 2000
431.	Imre Dobos	Production-inventory strategies for a linear reverse logistics system, Oktober 1999	447.	Anne Chwolka	"Marktorientierte Zielkostenvorgaben als Instrument der Verhaltenssteuerung im Kostenmanagement", März 2000
432.	Jan Wenzelburger	Convergence of Adaptive Learning in Models of Pure Exchange October 1999	448.	Volker Böhm, Carl Chiarella	Mean Variance Preferences, Expectations Formation, and the Dynamics of Random Asset Prices April 2000
433.	Imre Dobos, Klaus- Peter Kistner:	Production-inventory control in a reverse logistics system November 1999	449.	Beate Pilgrim	Non-equivalence of uniqueness of equilibria in complete and in incomplete market models, March 2000
434.	Joachim Frohn	The Foundation of the China-Europe-International-Business-School (CEIBS) November 1999	450.	Beate Pilgrim	A Brief Note on Mas-Colell's First Observation on Sunspots, March 2000
435.	Pu Chen, Joachim Frohn	Goodness of Fit Measures and Model Selection for Qualitative Response Models November 1999	451.	Thorsten Temme	An Integrated Approach for the Use of CHAID in Applied Marketing Research, May 2000
436.	Rolf König, Caren Sureth	Some new aspects of neoclassical investment theory with taxes, Dezember 1999	452.	Reinhold Decker, Claudia Bornemeyer	Ausgewählte Ansätze zur Entscheidungsunterstützung im Rahmen der Produktliniengestaltung, Mai 2000
437.	Rolf König, Elke Ohrem	The Effects of Taxation on the Dividend Behaviour of Corporations: Empirical Tests Dezember 1999	453.	Martin Feldmann	Threshold Accepting with a Back Step. Excellent results with a hybrid variant of Threshold Accepting, Mai 2000
438.	Jens-Ulrich Peter, Klaus Reiner Schenk- Hoppé	Business Cycle Phenomena in Overlapping Generations Economies with Stochastic Production November 1999	454.	Willi Semmler, Malte Sieveking	Credit Risk and Sustainable Debt: A Model and Estimations for Euroland November 1999
439.	Thorsten Temme, Reinhold Decker	CHAID als Instrument des Data Mining in der Marketingforschung Dezember 1999	455.	Alexander Krüger, Ralf-Michael	Der Euro - eine schwache Währung? Mai 2000
440.	Nicole Deutscher	Stock Market Equilibrium in OLG Economies with Heterogeneous			

Diskussionspapiere der Fakultät für Wirtschaftswissenschaften

Marquardt				
456.	Veith Tiemann	Symmetrische/klassische Kryptographie - Ein interaktiver Überblick, Mai 2000		
457.	Imre Dobos	Umweltbewusste Produktionsplanung auf Grundlage einer dynamischen umweltorientierten Produktionstheorie: Eine Projektbeschreibung Juni 2000	471.	Hermann Jahnke, Anne Chwolka, Dirk Simons Coordinating demand and capacity by adaptive decision making September 2001
458.	Imre Dobos	Optimal production-inventory strategies for a HMMS-type reverse logistics system Juli 2000	472.	Thorsten Pampel Approximation of generalized connecting orbits with asymptotic rate, September 2001
459.	Joachim Frohn	Ein Marktmodell zur Erfassung von Wanderungen (revidierte Fassung) Juli 2000	473.	Reinhold Decker Heiko Schimmelpfennig Assoziationskoeffizienten und Assoziationsregeln als Instrumente der Verbundmessung - Eine vergleichende Betrachtung, September 2001
460.	Klaus-Peter Kistner Imre Dobos	Ansätze einer umweltorientierten Produktionsplanung: Ergebnisse eines Seminars Juli 2000	474.	Peter Naeve Virtuelle Tabellensammlung, September 2001
461.	Reinhold Decker	Instrumentelle Entscheidungsunterstützung im Marketing am Beispiel der Verbundproblematik, September 2000	475.	Heinz-J. Bontrup Ralf-Michael Marquardt Germany's Reform of the Pension System: Choice between „Scylla and Charybdis“ Oktober 2001
462.	Caren Sureth	The influence of taxation on partially irreversible investment decisions - A real option approach, April 2000	476.	Alexander M. Krüger Wechselkurszielzonen zwischen Euro, Dollar und Yen -- nur eine Illusion? Oktober 2001
463.	Veith Tiemann	Asymmetrische/moderne Kryptographie - Ein interaktiver Überblick Oktober 2000	477.	Jan Wenzelburger Learning to predict rationally when beliefs are heterogeneous. Oktober 2001
464.	Carsten Köper	Stability Analysis of an Extended KMG Growth Dynamics December 2000	478.	Jan Wenzelburger Learning in linear models with expectational leads Oktober 2001
465.	Stefan Kardekewitz	Analyse der unilateralen Maßnahmen zur Vermeidung der Doppelbesteuerung im deutschen Erbschaftsteuerrecht Februar 2001	479.	Claudia Bornemeyer, Reinhold Decker Key Success Factors in City Marketing – Some Empirical Evidence - Oktober 2001
466.	Werner Glastetter	Zur Kontroverse über das angemessene wirtschafts- und konjunkturpolitische Paradigma – Einige Akzente der gesamtwirtschaftlichen Entwicklung Westdeutschlands von 1950 bis 1993 März 2001		Fred Becker Michael Tölle Personalentwicklung für Nachwuchswissenschaftler an der Universität Bielefeld: Eine explorative Studie zur Erhebung des Ist-Zustands und zur Begründung von Gestaltungsvorschlägen Oktober 2001
467.	Thomas Braun, Ariane Reiss	Benchmarkorientierte Portfolio-Strategien Mai 2001	480.	Dirk Biskup, Martin Feldmann On scheduling around large restrictive common due windows December 2001
468.	Martin Feldmann, Stephanie Müller	An incentive scheme for true information providing in SUPPLY CHAINS, Juni 2001	481.	Dirk Biskup A mixed-integer programming formulation for the ELSP with sequence-dependent setup-costs and setup-times December 2001
469.	Wolf-Jürgen Beyn, Thorsten Pampel, Willi Semmler	Dynamic optimization and Skiba sets in economic examples, August 2001	482.	Lars Grüne, Willi Semmler, Malte Sieveking Thresholds in a Credit Market Model with Multiple Equilibria August 2001
470.	Werner Glastetter	Zur Kontroverse über das angemessene wirtschafts- und konjunktur-	483.	Toichiro Asada Price Flexibility and Instability in a Macrodynamic Model with Debt Effect, February 2002

Diskussionspapiere der Fakultät für Wirtschaftswissenschaften

- | | | | | | |
|------|---|---|------|--|--|
| 484. | Rolf König,
Caren Sureth | Die ökonomische Analyse der Auswirkungen der Unternehmenssteuerreform auf Sachinvestitionsentscheidungen vor dem Hintergrund von Vorteilhaftigkeits- und Neutralitätsüberlegungen
- diskreter und stetiger Fall -
März 2002 | 497. | Jochen Jungeilges | On Chaotic Consistent Expectations Equilibria
March 2003 |
| 485. | Fred G. Becker,
Helge Probst | Personaleinführung für Universitätsprofessoren: Eine explorative Studie an den Universitäten in Nordrhein-Westfalen zum Angebot und an der Universität Bielefeld zum Bedarf
März 2002 | 498. | Volker Böhm | MACRODYN
- The Handbook -
March 2003 |
| 486. | Volker Böhm,
Tomoo Kikuchi | Dynamics of Endogenous Business Cycles and Exchange Rate Volatility
April 2002 | 499. | Jochen A. Jungeilges | Sequential Computation of Sample Moments and Sample Quantiles
– A Tool for MACRODYN -
April 2003 |
| 487. | Caren Sureth | Die Besteuerung von Beteiligungsveräußerungen - eine ökonomische Analyse der Interdependenzen von laufender und einmaliger Besteuerung vor dem Hintergrund der Forderung nach Rechtsformneutralität -
Juli 2002 | 500. | Fred G. Becker,
Vera Brenner | Personalfreisetzung in Familienunternehmen:
Eine explorative Studie zur Problematik
Juni 2003 |
| 488. | Reinhold Decker | Data Mining und Datenexploration in der Betriebswirtschaft
Juli 2002 | 501. | Michael J. Fallgatter,
Dirk Simons. | "Zum Überwachungsgefüge deutscher Kapitalgesellschaften - Eine anreiz-theoretische Analyse der Vergütung, Haftung und Selbstverpflichtung des Aufsichtsrates"
Juni 2003 |
| 489. | Ralf Wagner,
Kai-Stefan Beinke,
Michael Wendling | Good Odd Prices and Better Odd Prices - An Empirical Investigation
September 2002 | 502. | Pu Chen | Weak exogeneity in simultaneous equations models
Juli 2003 |
| 490. | Hans Gersbach,
Jan Wenzelburger | The Workout of Banking Crises: A Macroeconomic Perspective
September 2002 | 503. | Pu Chen | Testing weak exogeneity in VECM
Juli 2003 |
| 491. | Dirk Biskup,
Dirk Simons | Common due date scheduling with autonomous and induced learning
September 2002 | 504. | Fred G. Becker,
Carmen Schröder | Personalentwicklung von Nachwuchswissenschaftlern: Eine empirische Studie bei Habilitanden des Fachs "Betriebswirtschaftslehre"
Juli 2003 |
| 492. | Martin Feldmann,
Ralf Wagner | Navigation in Hypermedia: Neue Wege für Kunden und Mitarbeiter
September 2002 | 505. | Caren Sureth | Die Wirkungen gesetzlicher und theoretischer Übergangsregelungen bei Steuerreformen – eine ökonomische Analyse steuerinduzierter Verzerrungen am Beispiel der Reform der Besteuerung von Beteiligungserträgen -
August 2003 |
| 493. | Volker Böhm,
Jan Wenzelburger | On the Performance of Efficient Portfolios
November 2002 | 506. | Jan Wenzelburger | Learning to play best response in duopoly games"
August 2003 |
| 494. | J. Frohn, P. Chen,
W. Lemke,
Th. Archontakis,
Th. Domeratzki,
C. Flöttmann,
M. Hillebrand,
J. Kitanovic,
R. Rucha, M. Pullen | Empirische Analysen von Finanzmarktdaten
November 2002 | 507. | Dirk Simons | Quasirentenansätze und Lerneffekte
September 2003 |
| 495. | Volker Böhm | CAPM Basics
December 2002 | 508. | Dirk Simons
Dirk Biskup | Besteht ein Bedarf nach Dritthaftung des gesetzlichen Jahresabschlussprüfers?
September 2003 |
| 496. | Susanne Kalinowski,
Stefan Kardekewitz | Betriebstätte vs. Kapitalgesellschaft im Ausland - eine ökonomische Analyse
März 2003 | 509. | Tomoo Kikuchi | A Note on Symmetry Breaking in a World Economy with an International Financial Market.,
October 2003 |
| | | | 510. | Fred G. Becker
Oliver Krah | Explorative Studie zur Personaleinführung bei Unternehmen in OWL: Ergebnisübersicht
Oktober 2003 |
| | | | 511. | Martin Feldmann
Stephanie Müller | Simulation von Reentrant Lines mit ARENA: Ergebnisse eines Projektes |

Diskussionspapiere der Fakultät für Wirtschaftswissenschaften

		zur Betriebsinformatik Januar 2004	527.	Jan Wenzelburger Hans Gersbach	Risk Premia in Banking and the Macroeconomy" December 2004
512.	Xuemin Zhao Reinhold Decker	Choice of Foreign Market Entry Mode Cognitions from Empirical and Theoretical Studies January 2004	528.	Joachim Frohn, Chen Pu	Alternative ökonomische Zeitverwendungsmodelle Dezember 2004
513.	Volker Böhm Jochen Jungeilges	Estimating Affine Economic Models With Discrete Random Perturbations January 2004	529.	Stefan Niermann Joachim Frohn	Standortfaktoren und ihre Bedeutung für das Abwandern von Unternehmen
514.	Ralf Wagner	Mining Promising Qualification Patterns February 2004	530.	Christoph Wöster	Constructing Arbitrage-free Binomial Models December 2004
515.	Ralf Wagner	Contemporary Marketing Practices in Russia February 2004	531.	Fred G. Becker, Natascha Henseler u.a.	Fremdmanagement in Familienunternehmen Januar 2005
516.	Reinhold Decker Ralf Wagner Sören Scholz	Environmental Scanning in Marketing Planning – An Internet-Based Approach –	532.	Andreas Scholze	Die Bestimmung des Fortführungs- werts in der Unternehmensbewertung mithilfe des Residualgewinnmodells Februar 2005
517.	Dirk Biskup Martin Feldmann	Lot streaming with variable sublots: an integer programming formulation April 2004	533.	Marten Hillebrand Jan Wenzelburger	On the Dynamics of Asset Prices and Portfolios in a Multiperiod CAPM" February 2005
518.	Andreas Scholze	Folgebewertung des Geschäfts- oder Firmenswerts aus Sicht der Meß- bzw. Informationsgehaltsperspektive April 2004	534.	Jan Thomas Martini	Transfer Pricing for Coordination and Profit Determination: An Analysis of Alternative Schemes February 2005
519.	Hans Gersbach Jan Wenzelburger	Do risk premia protect from banking crises? May 2004	535.	Klaus Wersching	Innovation and Knowledge Spillover with Geographical and Technological Distance in an Agentbased Simulation Model May 2005
520.	Marten Hillebrand Jan Wenzelburger	The impact of multiperiod planning horizons on portfolios and asset prices in a dynamic CAPM May 2004			
521.	Stefan Wielenberg	Bedingte Zahlungsversprechen in der Unternehmenssanierung Juni 2004	536.	Anne Chwolka Jan Thomas Martini Dirk Simons	Accounting-Data-Based Transfer Prices in a Team-Investment Setting May 2005
522.	Sören Scholz, Ralf Wagner	The Quality of Prior Information Structure in Business Planning - An Experiment in Environmental Scanning - August 2004	537.	Sören W. Scholz Ralf Wagner	Autonomous Environmental Scanning on the World Wide Web June 2005
523.	Jan Thomas Martini Claus-Jochen Haake	Negotiated Transfer Pricing in a Team-Investment Setting October 2004	538.	Thorsten Pampel	On the convergence of balanced growth in continous time July 2005
524.	Reinhold Decker	Market basket analysis by means of a growing neural network November 2004	539.	Fred G. Becker Michael K. Ruppel	Karrierestau - Ein Problem von Führungskräften wie Organisationen Juli 2005
525.	Reinhold Decker Sören Scholz	Wie viel darf guter Service kosten? Einkaufsstättenbedingte Preiswahr- nehmung im Selbstmedikationsmarkt November 2004	540.	Li Xihao Jan Wenzelburger	Auction Prices and Asset Allocations of the Electronic Equity Trading System <i>Xetra</i> August 2005
526.	Fred G. Becker Roman Bobrichtchev Natascha Henseler	Ältere Arbeitnehmer und alternde Belegschaften: Eine empirische Studie bei den 100 größten deutschen Unternehmen Dezember 2004	541.	Volker Böhm Luca Colombo	Technology Choice with Externalities - A General Equilibrium Approach August 2005
			542.	Martin Feldmann Dirk Biskup	On lot streaming with multiple products August 2005

Diskussionspapiere der Fakultät für Wirtschaftswissenschaften

- | | | |
|------|------------------|---|
| 543. | Christoph Wöster | Die Ermittlung des Conversion
Factors im Futures-Handel
September 2005 |
| 544. | Thomas Braun | The impact of taxation on upper and
lower bounds of enterprise value
October 2005 |
| 545. | Christoph Wöster | Replication in Consistent Binomial
Models
November 2005 |